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SIMULATION OF THE BEAM-BEAM INTERACTION FOR PEP-II WITH UNEQUAL BEAM-BEAM PARAMETERS

Jennifer R. Eden† and Miguel A. Furman

Exploratory Studies Group, MS 71-H
Accelerator and Fusion Research Division
Lawrence Berkeley Laboratory
Berkeley, CA 94720

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ABSTRACT

We carry out a first systematic step in assessing, via multiparticle tracking simulations, the effects of the beam-beam interaction for the APIARY 6.3D and APIARY 7.5 designs of PEP-II when the nominal beam-beam parameters are not all equal. Specifically, we take two different approaches in breaking the equality of these parameters: In the first one, we set $\xi_{0x,+}= \xi_{0y,+}\equiv \xi_{0+}$ and $\xi_{0x,-}= \xi_{0y,-}\equiv \xi_{0-}$ with $\xi_{0+}\neq \xi_{0-}$. In the second, $\xi_{0x,+}= \xi_{0x,-}\equiv \xi_{0x}$ and $\xi_{0y,+}= \xi_{0y,-}\equiv \xi_{0y}$ with $\xi_{0x} \neq \xi_{0y}$. In both cases we maintain the pairwise equality of the rms beam sizes at the IP, and we keep the nominal luminosity fixed at its nominal value, $\mathscr{L}_0 = 3 \times 10^{33}$ cm^{-2} s^{-1}. Other constraints are in effect, as explained in the text. Parasitic collisions with nominal beam separation are included. In each approach there are different implications for bunch currents and emittances as the beam-beam parameters move away from full equality. These implications are spelled out but are not evaluated. We conclude that: (1) In both cases only the vertical beam blowup is significant, and this blowup behaves smoothly as the beam-beam parameters move away from full equality. (2) In the first approach, the dynamics favors $\xi_{0+}\approx 0.024$, $\xi_{0-}\approx 0.04$ over $\xi_{0+}= \xi_{0-}= 0.03$. (3) In the second, the dynamics favors $\xi_{0y} \approx 0.023$, $\xi_{0x} \approx 0.04$ over $\xi_{0x} = \xi_{0y} = 0.03$. In either case, the resultant value for the dynamical luminosity is ~10% higher than that corresponding to the fully-symmetric case, while the total current of the low-energy beam approaches 3 A. Finally, we present a conjecture for the behavior of the dynamics seen in the simulations.

1. Introduction

The PEP-II B factory design[1,2] has been specified in such a way that all four nominal beam-beam parameters are constrained to be equal. This specification is a particular choice for one of the conditions of transparency symmetry.[3,4] An important practical implication of this constraint is that it reduces considerably the parameter space and hence simplifies the design. Furthermore, this symmetry is generally thought to provide a prudent starting point for the design of asymmetric colliders since it makes the beam-beam dynamics resemble that of symmetric, single-ring colliders for which a body of experience exists. From the theoretical perspective, the situation is not

† Current e-mail address: eden@geophys.washington.edu.

completely settled: it has been argued, from general principles, that the global beam-beam limit (understood to mean maximum integrated luminosity at a fixed overall cost) in an asymmetric collider can only be reached under asymmetric conditions.[5] On the other hand, a single-particle hamiltonian analysis for round beams in the linear-lattice approximation leads to the conclusion that the beam-beam limit is reached under a rather stringent, symmetric, set of conditions.[6]

In any case, the design of the B factory must strike a compromise among competing requirements from different areas of the design. This compromise requires accommodating certain constraints, such as those arising from single-particle nonlinear dynamics, synchrotron radiation masking, etc., that may affect an idealized optimization of the beam-beam interaction. In fact, the current design of PEP-II does not rigorously satisfy any set of transparency conditions. Early simulation studies showed that a transparent-symmetric PEP-II design would have better luminosity performance than a design in which the symmetry is badly broken.[4] Since the current design breaks the transparency symmetry to some extent, it seems natural to examine gradual departures from full equality in the beam-beam parameters, since this equality that has so far been maintained.[1,2,4,7–12] We stress that tracking simulations for the nominal designs for APIARY 6.3D and APIARY 7.5, with $\xi_{0x,+}= \xi_{0y,+}= \xi_{0x,-}= \xi_{0y,-}= 0.03$, have shown acceptable dynamical luminosity, with $\mathscr{L}\sim 2.6 \times 10^{33}$ cm^{-2} s^{-1} for the former[7] and $\mathscr{L}\sim 2.8 \times 10^{33}$ cm^{-2} s^{-1} for the latter.[8] Thus we are not motivated by an improvement in this sense. Rather, we would like to explore whether the beam dynamics naturally favors other values for the beam-beam parameters. The conjecture is that, if these parameters were chosen according to the preference expressed by the dynamics, the operation of the machine would be smoother and its performance more reliable. Of course, there would be a cost associated with this potential improvement. Neither the cost or the potential increase in reliability are investigated in this note.

In this note we present a first systematic, although far from complete, assessment of the beam-beam effect on the luminosity performance of PEP-II for the interaction region (IR) designs APIARY 6.3D and APIARY 7.5 with unequal beam-beam parameters. Specifically, we take here two different approaches in breaking the equality of the beam-beam parameters: In approach A, we set $\xi_{0x,+}= \xi_{0y,+}\equiv \xi_{0+}$ and $\xi_{0x,-}= \xi_{0y,-}\equiv \xi_{0-}$ with $\xi_{0+}\neq \xi_{0-}$. In approach B, $\xi_{0x,+}= \xi_{0x,-}\equiv \xi_{0x}$ and $\xi_{0y,+}= \xi_{0y,-}\equiv \xi_{0y}$ with $\xi_{0x} \neq \xi_{0y}$. In both cases we maintain the pairwise equality of the rms beam sizes at the interaction point (IP), and we keep the nominal luminosity fixed at its nominal value, $\mathscr{L}_0 = 3 \times 10^{33}$ cm^{-2} s^{-1}. In approach B the transparency-symmetry constraint on the beam-beam parameters is respected,[4] but this is not the case in approach A. Other constraints are in effect, as explained below. We present our results in the form of plots of the beam-beam-induced beam blowup σ/σ_0 *vs.* ξ_{0+} (approach A) or *vs.* ξ_{0y} (approach B). Parasitic collisions (PCs) with nominal beam separation are included in these multiparticle simulation studies. In each approach there are different implications for bunch currents and emittances as the beam-beam parameters move away from full equality. These implications are spelled out but are not evaluated.

We conclude that: (1) In both approaches only the vertical beam blowup is significant, and this blowup behaves smoothly as the beam-beam parameters move away from full equality. (2) In the first approach, the dynamics favors $\xi_{0+}\approx 0.024$, $\xi_{0-}\approx 0.04$ over $\xi_{0+}= \xi_{0-}= 0.03$. (3) In the second, the dynamics favors $\xi_{0y} \approx 0.023$, $\xi_{0x} \approx 0.04$ over $\xi_{0x} = \xi_{0y} = 0.03$. In either case, the dynamical value of the luminosity is slightly increased from the values corresponding to $\xi_{0x,+}=\xi_{0y,+}=\xi_{0x,-}=\xi_{0y,-}=0.03$: for APIARY 6.3D the dynamical luminosity at the preferred values of the ξ_0 parameters is $\mathscr{L}\sim 3.1\times 10^{33}$ cm^{-2} s^{-1} and for APIARY 7.5 it is $\mathscr{L}\sim 2.9$–3.3×10^{33} cm^{-2} s^{-1} (the dynamical value can be larger than the nominal value of 3×10^{33} cm^{-2} s^{-1} on account of the "dynamical beta-function effect").

An optimization study along the lines presented here, and an evaluation of the design and cost implications of unequal beam-beam parameters, fall wholly outside the scope of this note.

2. Assumptions and constraints

All basic lattice and nominal beam parameters (for the case in which $\xi_{0x,+}=\xi_{0y,+}=\xi_{0x,-}=\xi_{0y,-}=0.03$) are listed in Table 1 (APIARY 6.3D) and Table 2 (APIARY 7.5). When the beam-beam parameters are not all equal, the actual values of other parameters vary according to the approach taken to break the equality of the beam-beam parameters, as explained below. Here are our assumptions:

In all cases presented here we have looked at only one working point,* namely $(\nu_x, \nu_y) = (0.64, 0.57)$ for both beams, following the results of previous tune scans.[2] We consider only the linear approximation to the lattice, which is therefore fully described by the tunes, the lattice functions at the IP and PCs, and the intervening phase advances. We imagine the lattice divided up into two symmetrical "short" arcs, from the IP to each of the two PCs, and one "long" arc, from one PC to the other. The lattice tune is set by adjusting the phase advance of the long arc; the phase advances $\Delta\nu$ of the short arcs are fixed.

The RF wavelength, λ_{RF}, is 0.6298 m, and we consider only the nominal value for the bunch spacing, namely $s_B = 2\lambda_{RF} = 1.2596$ m. As a result, the collision frequency f_c is also fixed at its nominal value of 238 MHz, and the first PC occurs at a distance $\Delta s = 0.6298$ m from the IP. The beam energy E, bunch length σ_ℓ, rms energy spread σ_E/E and synchrotron tune† ν_s are different for the two beams. On the other hand, the horizontal and vertical damping times are equal to each other and in the two beams. In the simulations we hold all these parameters fixed at their nominal values, stated in Tables 1 and 2.

We maintain the restriction that the nominal rms beam sizes at the IP should be pairwise equal, namely

$$\sigma_{0x,+}=\sigma_{0x,-}\equiv\sigma_{0x}, \qquad \sigma_{0y,+}=\sigma_{0y,-}\equiv\sigma_{0y} \tag{1}$$

* This is the so-called "bare lattice" working point. The simulations here do not involve any form of tune compensation.[12]

† For historical reasons that are now irrelevant, we use a value of 0.0403 for the synchrotron tune of the LEB. In fact, the CDR specifies a value of 0.05. In Ref. 8 it is shown that simulations with a value of 0.05 yields slightly better performance.

although their actual numerical values may vary away from those listed in Tables 1 and 2 when the beam-beam parameters are not all equal. The beta functions at the IP, however, remain fixed at their nominal values throughout these studies,

$$\begin{aligned} &\beta^*_{x+} = 37.5 \text{ cm}, \quad \beta^*_{x-} = 75.0 \text{ cm} \\ &\beta^*_{y+} = 1.5 \text{ cm}, \quad \beta^*_{y-} = 3.0 \text{ cm} \end{aligned} \tag{2}$$

As mentioned above, we have followed two approaches to break the equality of the beam-beam parameters, namely

$$\text{Approach A:} \quad \xi_{0x,+} = \xi_{0y,+} \equiv \xi_{0+}, \quad \xi_{0x,-} = \xi_{0y,-} \equiv \xi_{0-}, \quad \xi_{0+} \neq \xi_{0-} \tag{3a}$$

or

$$\text{Approach B:} \quad \xi_{0x,+} = \xi_{0x,-} \equiv \xi_{0x}, \quad \xi_{0y,+} = \xi_{0y,-} \equiv \xi_{0y}, \quad \xi_{0x} \neq \xi_{0y} \tag{3b}$$

In order to fully determine the primary set of four parameters σ_{0x}, σ_{0y}, N_+ and N_- under these constraints, it turns out[14] that we need three additional numerical inputs. We choose one of them to be the nominal luminosity, which we fix at its nominal value,

$$\mathcal{L}_0 = f_c \frac{N_+ N_-}{4\pi\sigma_{0x}\sigma_{0y}} = 3 \times 10^{33} \quad \text{cm}^{-2}\text{s}^{-1} \tag{4}$$

The remaining two inputs are the nominal beam-beam parameters ξ_{0+}, ξ_{0-} (Approach A) or ξ_{0x}, ξ_{0y} (Approach B). As mentioned above, we present our simulation results by plotting beam blowup against ξ_{0+} or ξ_{0y}, depending on the approach. This means that ξ_{0+} or ξ_{0y} is given in each run; therefore the only additional parameter that needs to be specified is ξ_{0-} or ξ_{0x}. We have adopted the prescription that this parameter is determined by

$$\xi_{0+} \cdot \xi_{0-} = 0.03^2 \tag{5a}$$

or

$$\xi_{0x} \cdot \xi_{0y} = 0.03^2 \tag{5b}$$

depending on the approach taken. It should be emphasized that there are no *a priori* physical reasons for this prescription. From the purely mathematical point of view, all algorithms that fix ξ_{0-} or ξ_{0x} are equivalent, including specifications "by hand." Of course, each algorithm entails different sets of beam dynamics results and different implications for the other beam parameters. At this stage of our studies, prescription (5) is an arbitrary but convenient choice; its only virtue is that it allows a smooth extrapolation away from the nominal case, $\xi_{0+} = \xi_{0-} = 0.03$ or $\xi_{0x} = \xi_{0y} = 0.03$.

For each set of values of the nominal beam-beam parameters we determine the resultant number of particles per bunch and nominal emittances, and we run a simulation. We use here the simulation code TRS,[13] whose details are explained in Ref. 1. In the cases presented here we have chosen 256 superparticles per bunch, divided into five slices in order to represent the thick lens effects in the beam-beam interaction. We have run the simulations for 25,000 turns, or about five damping times; the beam blowup is determined by averaging over the last 2,500 turns of the run. The code was run on a Cray-2S/8128 computer at NERSC. Under these conditions (256 superparticles per beam, 5 slices and 25,000 turns), each run takes ~22 CPU min, and the CPU time scales approximately linearly in any of these three variables in this parameter regime.

3. A note on transparency symmetry

In a basic form, transparency symmetry[4] consists of four conditions, namely

(i) pairwise equality of nominal beam-beam parameters: $\xi_{0x,+} = \xi_{0x,-}$ and $\xi_{0y,+} = \xi_{0y,-}$

(ii) pairwise equality of nominal beam sizes at the IP: $\sigma_{0x,+} = \sigma_{0x,-}$ and $\sigma_{0y,+} = \sigma_{0y,-}$

(iii) equality of damping times of the two rings: $\tau_{x+} = \tau_{x-}$ and $\tau_{y+} = \tau_{y-}$

(iv) equality of the tune modulation amplitudes due to synchrotron oscillations: $(\sigma_\ell \nu_s/\beta^*_x)_+ = (\sigma_\ell \nu_s/\beta^*_x)_-$ and $(\sigma_\ell \nu_s/\beta^*_y)_+ = (\sigma_\ell \nu_s/\beta^*_y)_-$

In our studies here conditions (ii) and (iii) are satisfied in both approaches. Condition (i) is satisfied in approach B but not in approach A. Even in approach B, however, transparency is broken because condition (iv) is not satisfied by the design: the parameters in Tables 1 or 2 violate these equalities at the ~40% level,

$$\begin{aligned} \left(\frac{\sigma_\ell \nu_s}{\beta_x^*}\right)_+ &= 1.0\times10^{-3}, \qquad \left(\frac{\sigma_\ell \nu_s}{\beta_x^*}\right)_- = 6.9\times10^{-4} \\ \left(\frac{\sigma_\ell \nu_s}{\beta_y^*}\right)_+ &= 2.6\times10^{-2}, \qquad \left(\frac{\sigma_\ell \nu_s}{\beta_y^*}\right)_- = 1.7\times10^{-2} \end{aligned} \tag{6}$$

Therefore, transparency symmetry is never exactly satisfied in these simulations, nor in most of the previous beam-beam simulation studies.[1,2,4,7–12]

4. Determination of the nominal beam sizes and currents in each approach

4.1 Approach A: $\xi_{0x} = \xi_{0y}$ *but* $\xi_{0+} \neq \xi_{0-}$

In this case it is easy to see that the inequality $\xi_{0+} \neq \xi_{0-}$ subject to the constraints $\xi_{0x} = \xi_{0y}$, $\mathscr{L}_0 = 3 \times 10^{33}$ cm^{-2} s^{-1} and $\xi_{0+} \cdot \xi_{0-} = 0.03^2$ is achieved by simply changing the numbers of particles per bunch at fixed nominal emittance in such a way that the product $N_+ \cdot N_-$ remains fixed at its nominal value, $N_+ \cdot N_- = 21.833 \times 10^{20}$. Thus in this approach the emittances remain constant at their nominal values as the beam-beam parameters vary away from 0.03. Of course, the PC-induced beam-beam parameters change because they depend on $N_\pm$

Explicitly, taking Eq. (5a) into account, the scaling relations are:

$$N_+ = 5.630 \times 10^{10} \times \left(\frac{0.03}{\xi_{0,+}}\right) \tag{7a}$$

$$N_- = 3.878 \times 10^{10} \times \left(\frac{\xi_{0,+}}{0.03}\right) \tag{7b}$$

$$\sigma_{0x} = \text{constant} = 185.6\ \mu\text{m} \tag{7c}$$

$$\sigma_{0y} = \text{constant} = 7.4\ \mu\text{m} \tag{7d}$$

The total beam currents resulting from the N's and the σ's are plotted in Fig. 3.

4.2 Approach B: $\xi_{0+} = \xi_{0-}$ *but* $\xi_{0x} \neq \xi_{0y}$

In this case the variation of the beam-beam parameters away from complete equality entails changes both in the number of particles per bunch and rms beam sizes. One finds[14] the following scaling formulas:

$$N_\pm = [N_\pm]_{\text{nom.}} \times f(\xi_{0y}) \tag{8a}$$

$$\sigma_{0x} = [\sigma_{0x}]_{\text{nom.}} \times \left(\frac{\xi_{0y}}{0.03}\right) \times f(\xi_{0y}) \tag{8b}$$

$$\sigma_{0y} = [\sigma_{0y}]_{\text{nom.}} \times \left(\frac{0.03}{\xi_{0y}}\right) \times f(\xi_{0y}) \tag{8c}$$

where the quantities in square brackets with the subscript "nom." are the nominal design values, *i.e.*, those corresponding to $\xi_{0x} = \xi_{0y} = 0.03$ (Tables 1 or 2), and $f(\xi_{0y})$ is the function

$$f(\xi_{0y}) \equiv \frac{1 + r_\beta}{\dfrac{\xi_{0y}}{0.03} + \dfrac{0.03}{\xi_{0y}} r_\beta} \tag{9}$$

where r_β is the beta-function ratio,

$$r_\beta \equiv \left(\frac{\beta_y^*}{\beta_x^*}\right)_+ = \left(\frac{\beta_y^*}{\beta_x^*}\right)_- \tag{10}$$

Again, the total beam currents resulting from the N's and the σ's are plotted in Fig. 3. Now since the beta function ratio is quite small for PEP-II ($r_\beta = 0.04$), the function $f(\xi_{0y})$ can be approximated by

$$f(\xi_{0y}) \approx \frac{0.03}{\xi_{0y}} \tag{11}$$

with an accuracy better than ~5% over the range of values of interest for the beam-beam parameter, namely $0.02 \le \xi_{0y} \le 0.05$. Thus we find the following approximate scaling relations:

$$N_\pm \approx [N_\pm]_{\mathrm{nom.}} \times \frac{0.03}{\xi_{0y}} \tag{12a}$$

$$\sigma_{0x} \approx \mathrm{constant} = [\sigma_{0x}]_{\mathrm{nom.}} \tag{12b}$$

$$\sigma_{0y} \approx [\sigma_{0y}]_{\mathrm{nom.}} \times \left(\frac{0.03}{\xi_{0y}}\right)^2 \tag{12c}$$

5. Discussion of the results

We now compare the results for the four cases by looking at the plots for the beam blowup σ/σ_0 *vs.* ξ_{0+} (Approach A) or *vs.* ξ_{0y} (Approach B) for both designs. As mentioned earlier, we keep the PC separation fixed at its nominal value, d = 2.82 mm for APIARY 6.3D or d = 3.5 mm for APIARY 7.5.

Figure 1 shows the simulation results for beam blowup in approach A for both designs. Figure 2 shows the corresponding results for approach B. In the plots the arrow labeled "nominal" indicates the situation corresponding to $\xi_{0x,+} = \xi_{0y,+} = \xi_{0x,-} = \xi_{0y,-} = 0.03$. Both sets of results show that the horizontal beam dynamics is not sensitive to the choice of beam-beam parameters within the range of values we have chosen. The vertical dynamics, on the other hand, clearly prefers unequal beam-beam parameters.

In approach A, the vertical beam blowup for APIARY 6.3D shows a minimum at $\xi_{0+} \approx 0.023$ which corresponds, according to Eq. (5a), to $\xi_{0-} \approx 0.039$. Likewise, APIARY 7.5 shows a minimum at $\xi_{0+} \approx 0.026$. We do not understand the spikes in the blowup curves at $\xi_{0+} \approx 0.021$; we conjecture that these are caused by the excitation of a resonance. We do not understand why the vertical blowup curve for APIARY 7.5 comes down to almost unity (*i.e.*, almost nominal behavior) at $\xi_{0+} = 0.020$; for this value of ξ_{0+} the HEB experiences a beam-beam parameter $\xi_{0-} = 0.045$, which seems rather substantial. In approach B (Fig. 2), APIARY 6.3D prefers $\xi_{0y} \approx 0.023$, $\xi_{0x} \approx 0.039$ and APIARY 7.5 prefers $\xi_{0y} \approx 0.024$, $\xi_{0x} \approx 0.038$.

As mentioned in Section 4, departures from full equality of the nominal beam-beam parameters entail consequences for the currents and nominal emittances. Figure 3 shows the total beam currents and nominal rms beam sizes at the IP plotted against ξ_{0+} or ξ_{0y}, depending on the approach. In translating the number of particles per bunch into total beam current we have assumed that the beams have no gaps, *i.e.*, we have assumed that each beam has 1746 identical bunches with $N_{\pm}$given by Eqs. (7a-b) or (8a). The results for the N's and the σ_0's are the same for either design, APIARY 6.3D or APIARY 7.5.

In approach A the rms beam sizes are constant when ξ_{0+} varies, as stated in Eqs. (7c-d), and the HEB and LEB currents are proportional and inversely proportional to ξ_{0+}, respectively. These quantities are shown in the two left-hand side plots in Fig. 3. In approach B both the nominal rms beam sizes and currents have nontrivial dependence on ξ_{0y}, as stated in Eqs. (8). The set of Eqs. (8) (not the approximations (12)) are plotted in the two right-hand side plots in Fig. 3.

Tables 3 and 4 summarize the results of modifying the designs of APIARY 6.3D and APIARY 7.5 in such a way as to accommodate the above-mentioned preference expressed by the dynamics seen in the simulations. Also listed is the dynamical luminosity in each case.

TABLE 3: Modified nominal beam-beam parameters, rms beam sizes at the IP and total beam current in approach A, along with estimated dynamical luminosity.

	APIARY 6.3D		APIARY 7.5	
	LEB (e^+)	HEB (e^-)	LEB (e^+)	HEB (e^-)
ξ_{0x}	0.023	0.039	0.026	0.035
ξ_{0y}	0.023	0.039	0.026	0.035
σ_{0x} [µm]	186	186	186	186
σ_{0y} [µm]	7.43	7.43	7.43	7.43
$r \equiv \sigma_{0y}/\sigma_{0x}$	0.04		0.04	
I [A]	2.8	1.1	2.5	1.3
$\mathscr{L}$ [cm^{-2} s^{-1}]	$\sim 3.1 \times 10^{33}$		$\sim 3.3 \times 10^{33}$	

TABLE 4: Modified nominal beam-beam parameters, rms beam sizes at the IP and total beam current in approach B, along with estimated dynamical luminosity.

	APIARY 6.3D	APIARY 7.5

	LEB (e^+)	HEB (e^-)	LEB (e^+)	HEB (e^-)
ξ_{0x}	0.039	0.039	0.038	0.038
ξ_{0y}	0.023	0.023	0.024	0.024
σ_{0x} [μm]	181	181	182	182
σ_{0y} [μm]	12.3	12.3	11.4	11.4
$r \equiv \sigma_{0y}/\sigma_{0x}$	0.068		0.063	
I [A]	2.7	1.9	2.6	1.8
$\mathscr{L}$ [cm^{-2} s^{-1}]	$\sim 3.1\times 10^{33}$		$\sim 2.9\times 10^{33}$	

6. A possible qualitative explanation

In order to try to gain some qualitative understanding of the vertical blowup plots in Fig. 1, we have tried to correlate them with the total nominal beam-beam parameters, which are plotted in Fig. 4. Each one of the four total nominal beam-beam parameter is defined to be

$$\xi_{0,\mathrm{tot}} \equiv \xi_0 + 2\xi_{0,\mathrm{PC}} \tag{13}$$

where ξ_0 is the nominal beam-beam parameter at the IP (*e.g.*, any of the parameters ξ_{0+}, ξ_{0-}, ξ_{0y}, or ξ_{0x}) and $\xi_{0,\mathrm{PC}}$ is the beam-beam parameter induced by the first parasitic collision. $\xi_{0,\mathrm{tot}}$ is the total beam-beam parameter experienced by a particle at the center of the bunch in one turn. The factor 2 in front of $\xi_{0,\mathrm{PC}}$ accounts for the fact that each bunch experiences two PCs per turn, one on either side of the IP.

In approach A, as seen in the two left-hand side plots in Fig. 4, the two total beam-beam parameters of the LEB (solid and dotted lines) are proportional to ξ_{0+}, while those of the HEB (dashed and dot-dashed lines) are inversely proportional to ξ_{0+}. Because of the opposite functional dependence, the curves necessarily cross. The crossing point corresponds to the situation where neither beam is "strong" or "weak," *i.e.*, the two beams are balanced. By equating the expressions for $\xi_{0y+,\mathrm{tot}}$ and $\xi_{0y-,\mathrm{tot}}$ (intersection of the dotted and dot-dashed lines) we find an expression for the balance point for the vertical dynamics,

$$\xi_{0+,\mathrm{bal}} = \left[\xi_0\right]_{\mathrm{nom}} \times \sqrt{\left[\frac{\xi_{0y-,\mathrm{tot}}}{\xi_{0y+,\mathrm{tot}}}\right]_{\mathrm{nom}}} \tag{14}$$

where the quantities inside the square brackets with the subscript "nom" are those that correspond to $\xi_{0+} = \xi_{0-} = 0.03$. For APIARY 6.3D we obtain, from Table 1,

$$\text{APIARY 6.3D:} \quad \xi_{0+,\mathrm{bal}} = 0.03 \times \sqrt{\frac{0.0347}{0.0482}} = 0.0255 \tag{15}$$

while for APIARY 7.5 we obtain, from Table 2,

$$\text{APIARY 7.5:} \qquad \xi_{0+,\text{bal}} = 0.03\times\sqrt{\frac{0.0331}{0.0424}} = 0.0265 \qquad (16)$$

These balance points correlate well with the minima in the vertical blowup curves exhibited by the simulation results in Fig. 1. It seems reasonable to conjecture that, in general, the balance point would yield optimum performance, under the approximations in effect in our studies.

We must point out that, in this approach, the fact that $\xi_{0+,\text{bal}}$ is different from the nominal value 0.03 is due to the inequality of the beta functions at the IP. If the design were such that $\beta^*_{y,+} = \beta^*_{y,-}$ then the beta functions at the PC would also be equal.* Consequently $\xi_{0y+,\text{tot}} = \xi_{0y-,\text{tot}}$ would obtain, and Eq. (14) would then yield $\xi_{0+,\text{bal}} = 0.03$.

In approach B the total vertical beam-beam parameters for the two beams scale together, and do not cross. However, the total vertical and horizontal beam-beam parameters for each beam do cross, and it seems reasonable that this crossing should be declared the balance point within this approach. A calculation for the LEB, using the approximation given by Eq. (11) for $f(\xi_{0y})$ yields, for the equality of $\xi_{0y+,\text{tot}}$ and $\xi_{0x+,\text{tot}}$ (intersection of the dotted and solid lines),

$$\xi_{0y,\text{bal}} \approx \sqrt{\left[\xi_0\left(\xi_0 + 2\xi_{0x+,\text{PC}} - 2\xi_{0y+,\text{PC}}\right)\right]_{\text{nom}}} \qquad (17)$$

By using the entries from Tables 1 and 2 we obtain the numerical values

$$\text{APIARY 6.3D:} \qquad \xi_{0y,\text{bal}} \approx \sqrt{0.03\times\left(0.03 + 2\times(-0.000544 - 0.009097)\right)} = 0.0179 \quad (18)$$

$$\text{APIARY 7.5:} \qquad \xi_{0y,\text{bal}} \approx \sqrt{0.03\times\left(0.03 + 2\times(-0.000336 - 0.006200)\right)} = 0.0225 \quad (19)$$

which, again, correlate well with the minima in the vertical blowup curves for the simulations for approach B shown in Fig. 1. In this approach, the balance point would be nontrivial (*i.e.*, different from 0.03) even if $\beta^*_{y,+} = \beta^*_{y,-}$.

7. Conclusions

We have carried out only two studies of departures from full equality of the beam-beam parameters for PEP-II. We have constrained the two approaches so that there is only one free parameter, and we have not attempted any sort of optimization along these lines. In particular, we have maintained the beta functions at the IP at their nominally-specified values.

Because of the nature of the approximations involved in these studies, we cannot guarantee the validity of the quantitative details of the results in Figs. 1 and 2. Nevertheless it seems clear, from the overall qualitative features of the results, that the beam dynamics prefers unequal over equal beam-beam parameters.

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* This would be exactly true for APIARY 7.5 but only approximately true for APIARY 6.3D.

Under the constraints of approach A, the preference is for a smaller beam-beam parameter for the LEB than for the HEB. We conjecture that this preference is probably due to the inequality of the PC-induced beam-beam parameters. This inequality, in turn, is a consequence of the inequality of the beta functions at the IP, $\beta^*_{y,+} \neq \beta^*_{y,-}$. If the PEP-II design were changed within the constraints of this approach in order to satisfy the preference expressed by the beam-beam dynamics, the total beam currents would need to be ~1.1–1.3 A and ~2.5–2.8 A for the HEB and LEB, respectively, and the nominal rms beam sizes would remain unaltered from the nominally-specified values.

Under the constraints of approach B, the preference is for a vertical beam-beam parameter that is smaller than the horizontal. This result is qualitatively consistent with the operational experience of existing colliders such as CESR.[15] If the PEP-II design were changed within the constraints of this approach in order to satisfy this preference, the total beam currents would be ~1.8–1.9 A and ~2.6–2.7 A for the HEB and LEB, respectively, and the nominal rms beam sizes at the IP would be σ_{0y} ~11–12 μm and σ_{0x} ~181–182 μm. We conjecture that this preference would still hold true in a more symmetric case, with $\beta^*_{y,+} = \beta^*_{y,-}$.

In either approach, the total current of the LEB becomes significantly larger than its value of 2.15 A for the fully-symmetric case ($\xi_{0x,+} = \xi_{0y,+} = \xi_{0x,-} = \xi_{0y,-} = 0.03$) at the point preferred by the dynamics. And, in approach B, the HEB current approaches 2 A, which should be compared to 1.5 A in the fully-symmetric case. The estimate for the dynamical luminosity from the simulations is $\mathscr{L}$ Ö3 $\times 10^{33}$ cm^{-2} s^{-1}. It is slightly larger than the nominal value, $\mathscr{L}_0 = 3 \times 10^{33}$ cm^{-2} s^{-1}, on account of the dynamical beta-function effect: a calculation shows that all four dynamical beta functions at the IP are smaller than the nominal ones for the working point we have chosen.[16] If the emittances did not blow up, or if they blew up by a small amount, $\mathscr{L}$ would be dominated by the dynamical beta-function effect, and would be larger than $\mathscr{L}_0$. This is undoubtedly the explanation of our simulation results.

8. Acknowledgements

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TABLE 1

APIARY 6.3D PRIMARY PARAMETERS
Nominal CDR case; $\mathscr{L}_0 = 3 \times 10^{33}$ cm^{-2} s^{-1}; $\xi_0 = 0.03$

	LER (e^+)	HER (e^-)
$\mathscr{L}_0$ [cm^{-2} s^{-1}]	3×10^{33}	
C [m]	2199.32	2199.32
E [GeV]	3.1	9.0
s_B [m]	1.2596	1.2596
f_c [MHz]	238.000	
V_{RF} [MV]	8.0	18.5
f_{RF} [MHz]	476.000	476.000
ϕ_s [deg]	170.6	168.7
α	1.15×10^{-3}	2.41×10^{-3}
ν_s	0.0403	0.0520
σ_ℓ [cm]	1.0	1.0
σ_E/E	1.00×10^{-3}	0.616×10^{-3}
N	5.630×10^{10}	3.878×10^{10}
I [A]	2.147	1.479
ε_{0x} [nm-rad]	91.90	45.95
ε_{0y} [nm-rad]	3.676	1.838
β^*_x [m]	0.375	0.750
β^*_y [m]	0.015	0.030
σ^*_{0x} [μm]	185.6	185.6
σ^*_{0y} [μm]	7.426	7.426
τ_x [turns]	5,014	5,014
τ_y [turns]	5,014	5,014

TABLE 1 (contd.)

APIARY 6.3D IP AND PC PARAMETERS
Nominal CDR case; $\mathcal{L}_0 = 3 \times 10^{33}$ cm^{-2} s^{-1}; $\xi_0 = 0.03$

	LER (e^+)		HER (e^-)	
Δs [cm] [a]	62.9816			
d [mm] [a]	2.82			
	IP	1st PC	IP	1st PC
$\Delta\nu_x$ [a]	0	0.1643	0	0.1111
$\Delta\nu_y$ [a]	0	0.2462	0	0.2424
β_x [m]	0.375	1.51	0.750	1.30
β_y [m]	0.015	25.23	0.030	13.01
α_x	0	–2.42	0	–1.06
α_y	0	–29.25	0	–18.74
σ_{0x} [μm]	185.6	372.4	185.6	244.4
σ_{0y} [μm]	7.426	304.5	7.426	154.6
$\sigma_{0x'}$ [mrad]	0.495	0.646	0.248	0.274
$\sigma_{0y'}$ [mrad]	0.495	0.353	0.248	0.223
d/σ_{0x}	0	7.570	0	11.538
ξ_{0x}	0.03	–0.000544	0.03	–0.000234
ξ_{0y}	0.03	+0.009097	0.03	+0.002345
$\xi_{0x,tot}$ [b]	0.0289		0.0295	
$\xi_{0y,tot}$ [b]	0.0482		0.0347	

[a] The first PC occurs at a distance Δs and at a phase advance $\Delta\nu$ from the IP. At this point the nominal orbits are separated horizontally by a distance d.

[b] The total nominal beam-beam parameter is defined to be $\xi_{0,tot} \equiv \xi_0^{(\mathrm{IP})} + 2\xi_0^{(\mathrm{PC})}$.

TABLE 2

APIARY 7.5 PRIMARY PARAMETERS
Nominal DU case; $\mathscr{L}_0 = 3 \times 10^{33}$ cm^{-2} s^{-1}; $\xi_0 = 0.03$

	LER (e^+)	HER (e^-)
$\mathscr{L}_0$ [cm^{-2} s^{-1}]	3×10^{33}	
C [m]	2199.32	2199.32
E [GeV]	3.1	9.0
s_B [m]	1.2596	1.2596
f_c [MHz]	238.000	
V_{RF} [MV]	8.0	18.5
f_{RF} [MHz]	476.000	476.000
ϕ_s [deg]	170.6	168.7
α	1.15×10^{-3}	2.41×10^{-3}
ν_s	0.0403	0.0520
σ_ℓ [cm]	1.0	1.0
σ_E/E	1.00×10^{-3}	0.616×10^{-3}
N	5.630×10^{10}	3.878×10^{10}
I [A]	2.147	1.479
ε_{0x} [nm-rad]	91.90	45.95
ε_{0y} [nm-rad]	3.676	1.838
β^*_x [m]	0.375	0.750
β^*_y [m]	0.015	0.030
σ^*_{0x} [μm]	185.6	185.6
σ^*_{0y} [μm]	7.426	7.426
τ_x [turns]	5,014	5,014
τ_y [turns]	5,014	5,014

TABLE 2 (contd.)

APIARY 7.5 IP AND PC PARAMETERS
Nominal DU case; $\mathcal{L}_0 = 3 \times 10^{33}$ cm^{-2} s^{-1}; $\xi_0 = 0.03$

	LER (e^+)		HER (e^-)	
Δs [cm] [a]	62.9816			
d [mm] [a]	3.498			
	IP	1st PC	IP	1st PC
$\Delta\nu_x$ [a]	0	0.1645	0	0.1112
$\Delta\nu_y$ [a]	0	0.2462	0	0.2424
β_x [m]	0.375	1.433	0.750	1.279
β_y [m]	0.015	26.46	0.030	13.25
α_x	0	–1.680	0	–0.840
α_y	0	–41.988	0	–20.994
σ_{0x} [μm]	185.6	362.9	185.6	242.4
σ_{0y} [μm]	7.426	311.9	7.426	156.1
$\sigma_{0x'}$ [mrad]	0.495	0.495	0.248	0.248
$\sigma_{0y'}$ [mrad]	0.495	0.495	0.248	0.248
d/σ_{0x}	0	9.639	0	14.429
ξ_{0x}	0.03	–0.000336	0.03	–0.000150
ξ_{0y}	0.03	+0.006200	0.03	+0.001553
$\xi_{0x,tot}$ [b]	0.0293		0.0297	
$\xi_{0y,tot}$ [b]	0.0424		0.0331	

[a] The first PC occurs at a distance Δs and at a phase advance $\Delta\nu$ from the IP. At this point the nominal orbits are separated horizontally by a distance d.

[b] The total nominal beam-beam parameter is defined to be $\xi_{0,tot} \equiv \xi_0^{(IP)} + 2\xi_0^{(PC)}$.